Four-State Electronic Random Number Generator

This study provided an opportunity to determine whether the remote sensing capability could be extended to the perception of the internal state of a piece of electronic equipment. For this purpose, an automated experiment designed around a four-state electronic random number generator was initiated. The solid-state machine has no moving parts and provides no sensory cue to the user as to its target generation.

In order to determine unambiguously whether a result was meaningful, the following strategy was used. First, the randomness of the machine was verified by over 10,000 pre-experiment trials (details given below). Second, the subjects interacted with the machine to generate the data. Third, for any subject whose score was significant, the statistics of the machine during the successful experiment were tabulated to insure that the machine had not departed from randomness in the period in which a significant result was obtained. Fourth, a subject generating a good score was asked to repeat the entire experiment after a one-month lag period. Finally, the entire data analysis was carried out by an independent statistics group at SRI. (Dr. Richard Singleton).

The machine configuration provides as a target one of four art slides chosen randomly (p = 1/4) by an electronic random generator. The generator does not indicate its choice until the subject indicates his choice to the machine by pressing a button (see Figure 11). (The

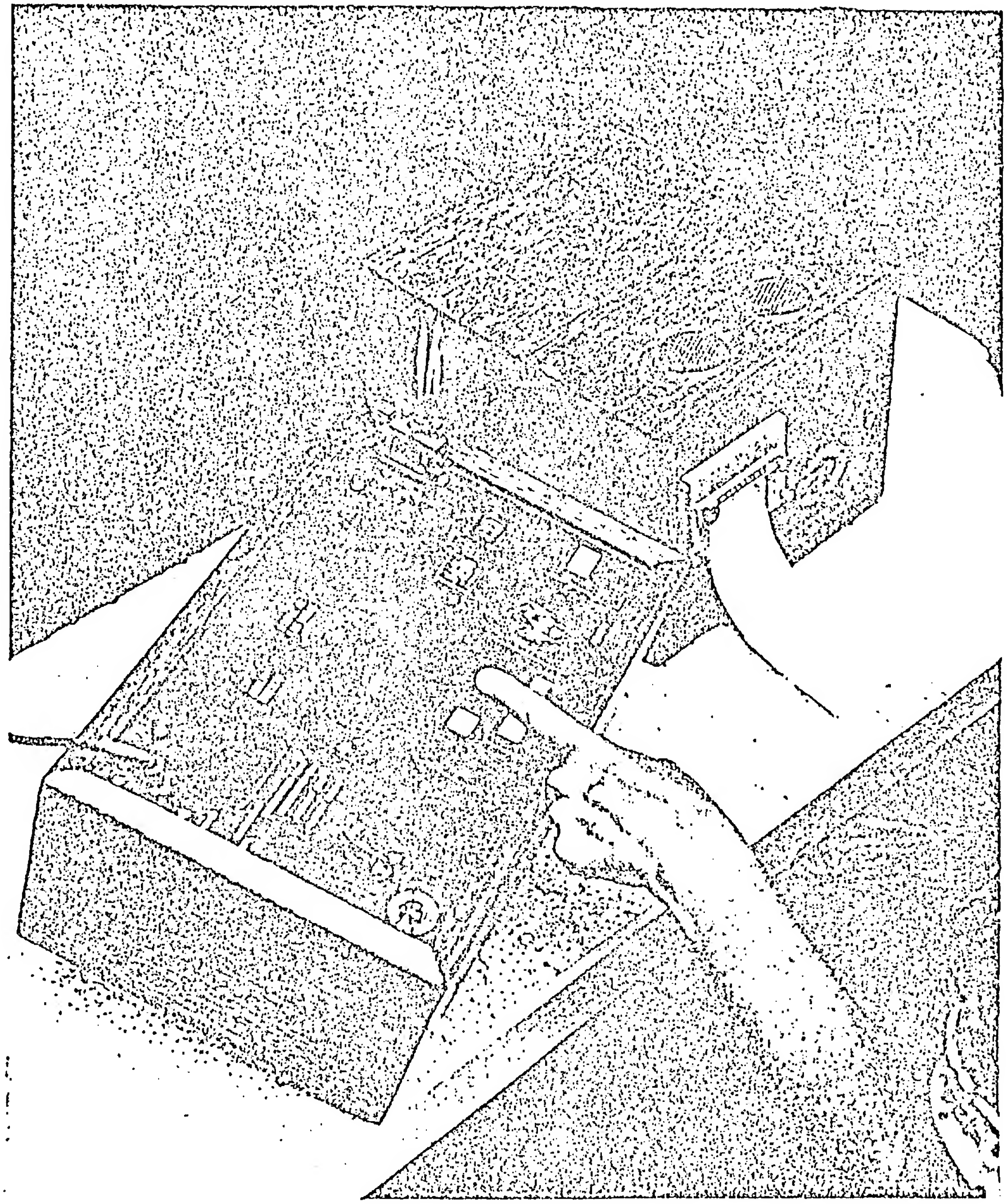

FIGURE 11 Four-state electronic random number generator used in this experiment. An incorrect choice of target is indicated. Two of the five "encouragement lights" at the top of the machine are illuminated. The printer to the right of the machine records data on fan-fold paper tape.

machine has four stable internal states. A 1.0-MHz square-wave oscillator sends pulses to an electronic "scale-of-four" counter which passes through each of its four states 250,000 times per second. The state of the counter is determined by the length of time the oscillator has run, that is, the time between subject choices.) As soon as the subject indicates his choice, the target slide is illuminated to provide visual and auditory (bell if correct) feedback as to the correctness or incorrectness of his choice. Until that time, both subject and experimenter remain ignorant of the machine's choice, so the experiment is of the double-blind type. Five legends at the top of the machine face are illuminated one at a time with increasing correct choices (6, 8, 10, ...) to provide additional reinforcement. The machine choice, subject choice, cumulative trial number, and cumulative hit number are recorded automatically on a printer. Following trial number 25, the machine must be reset manually by depressing a RESET button.

A methodological feature of the machine is that the choice of a target is not forced. That is, a subject may press a PASS button when he wishes not to guess, in which case the machine indicates what its choice was. The machine thus scores neither a hit nor a trial and then goes on to make its next selection. Thus, the subject does not have to guess at targets when he does not feel that he has an idea as to which to choose.

Under the null hypothesis of random binomial choices with probability 1/4 and no learning, the probability of observing $\geq k$ successes in n trials is approximated by the probability of a normal distribution value,

$$\geq \left(k - \frac{n}{4} - \frac{1}{2}\right) \Big/ \sqrt{3n/16}$$

Pre-Experiment Randomness Tests

The design objective was to build a four-state machine, with each state equally likely to occur on each trial, independent of the past sequence of states. If the machine meets this objective, it should not be possible to devise a rule for future play that significantly differs from chance. A simple example of such a rule would be to select the machine state observed in the preceding trial; if this strategy were to produce scores significantly above chance (25 percent hits), we would reject the hypothesis of randomness of the machine under test.

Before experimentation machines purchased from Aquarius Electronics, Albion, California, were extensively tested for randomness. Data were analyzed on a CDC-6400 computer, and the machine finally selected for use met established criteria for randomness.

In developing randomness tests, we are guided in part by a knowledge of the machine logic. When one of the four choice keys or the pass key is depressed, the current machine state is displayed; then a brief time after release of the key, a new machine state is established (but not shown to the subject) by sampling the instantaneous state of a highspeed four-state electronic counter. For the machine to be random, the times of dwell of the counter in each of the four states must be precisely equal; otherwise, the distribution of outcomes will be biased. The first randomness test is thus based on tallying the number of occurrences of each of the four states. This test should detect a stable bias, yet may miss a drifting bias. To test for this second possibility we also tally the distribution of outcomes in each group of 100 trials, then compute a likelihood ratio test statistic (see below) for each group. Under the null hypothesis of equal likelihood of the four states, these statistic values are distributed approximately as chi-square with three degrees of freedom and their sum for m groups distributed approximately as chi-square with three m degrees of freedom. This test may also detect stable bias, but is not as powerful for this purpose as the first test. Variable bias of still a shorter period, if substantial, can be tested for by tallying the frequency with which the previous machine state is repeated; an overall repeat ratio ("all") significantly above 0.25 is indicative of such bias.

If for any reason the machine were to fail to sample the counter to establish a new state, the previous machine state would be repeated. To test for this possibility, we tally the number of repeats following the depression of each key. A repeat ratio significantly greater than 0.25 should be considered a danger signal.

We also tally the initial machine states following reset and the transitions between states. In each case, the number of occurrences of each of the four possible outcomes should be approximately equal. When repeats are deleted from the sequence of trials ("nondiagonal transitions"), the four states should also be approximately equal in frequency.

In testing the null hypothesis of four equally likely outcomes of a trial, a likelihood ratio test is used. The statistic

$$-2\sum_{i=1}^{4} n_i \,\ell n \left(\frac{n/4}{n_i}\right)$$

under the null hypothesis is distributed approximately as chi-square with three degrees of freedom, with rejection for large values of this statistic.* The computer program used in testing randomness includes a subroutine for computing the probability of a chi-square value as large or larger than that observed.

In testing the null hypothesis that the probability of a repeat is 0.25, the binomial probability of obtaining the observed number K or more repeats in N trials is computed. For K greater than 1000, a normal distribution approximation is computed, assuming the statistic

$$\left(\frac{K - 1/2}{N} - 0.25\right) \sqrt{\frac{N}{3/16}}$$

to be approximately normal with mean zero and standard deviation one.

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*Alexander Mood, Introduction to the Theory of Statistics (McGraw Hill, New York, 1950).

The typical test pattern used was six passes followed by 25 choices of one color, repeating this for each of the four colors. In this way each of the five keys other than reset were given approximately equal use. Typically, 2000 to 6000 trials were made in each sitting. In the absence of any unusual results in the randomness tests, a minimum of 10,000 trials were made before using a machine with experimental subjects. With 10,000 trials, the expected fraction of repeats is 0.25 with a standard deviation of 3/200 = 0.00866.

A computer listing of the results of randomness tests is included in Table 1. No significant departures from randomness were observed.

Subject Data

Data was collected from subjects S1 through S6. Each subject was asked to complete 100 25-trial runs (i.e., a total of 2500 trials each). The results are tabulated in Table 2. (One subject, S3, declined to complete the 2500-trial run, indicating a lack of rapport with the machine and, hence, a lack of motivation for the task.) For the six subjects, only one (S2) scored significantly above chance. For the 2500 trials that subject averaged 29.36 hits/100 trials rather than the expected 25/100, a result whose *a priori* probability under the null hypothesis is $p = 3 \times 10^{-7}$. His scores are plotted in Figure 12.

The statistics of the machine during the successful run of subject S2 were tabulated for the entire 3488 machine transitions (2500 choices, passes). The results, shown in Table 3, indicate no significant departure from random expectation during the successful run, and therefore, the significant result cannot be attributed to machine malfunction.

At a later time, subject S2 was asked to repeat the entire experiment, and he was able to replicate successfully a high mean scoring rate (27.88/100 average over 2500 trials, a result whose *a priori* probability under the null hypothesis is $p = 4.8 \times 10^{-4}$).

We thus conclude from this part of the study that of the six subjects tested, one subject (S2) generated a significant result replicable and not attributable to machine malfunction.

Finally, the study taken as a whole (15,750 trials) was significant, yielding an average scoring rate 26.47 hits/100 trials, a result whose *a priori* probability under the null hypothesis is $p = 1.1 \times 10^{-5}$.

The bit rate associated with the information channel can be calculated from

$$R = H(x) - H_y(x) \quad ,$$

where $H(x)$ is the uncertainty of the source message containing symbols with *a priori* probability P_i

$$H(x) = \sum_{i=1}^{4} P_i \log_2 P_i$$

and $H_y(x)$ is the conditional entropy based on the *a posteriori* probabilities that a received symbol was actually transmitted

$$H_y(x) = - \sum_{i,j=1}^{4} P(i,j) \log_2 P_i(j).$$

For S2's first run, with $P_i = 1/4$, $P(k,k) = 0.2936$, and an average of 30 seconds per choice, we have a source uncertainty $H(x) = 2$ bits and a calculated bit rate

$$R \approx 0.007 \text{ bits/symbol}$$

or

$$R/T \approx 2\text{x}10^{-4} \text{ bits/sec.}$$

Table 1

PRE-EXPERIMENT RANDOMNESS TESTS

	Buttons				Number of Trials	Chi-Sq.	Binom. Prob.
	Yellow	Green	Blue	Red			
Initial states	107	116	113	128	464	1.996	0.57
Transitions From/To Y	728	764	765	790	3047	2.573	0.46
G	777	784	773	863	3197	6.745	0.08
B	776	796	810	773	3155	1.158	0.76
R	787	852	803	805	3247	2.877	0.41
All states	3175	3312	3264	3359	13110	5.667	0.18
Nondiagonal transitions	2340	2412	2341	2426	9519	2.630	0.45
Diagonal transitions	728	784	810	805	3127	5.414	0.15

	Key	N-Trials	Repeats	Ratio	Bionomial Prob.
Diagonal transitions as a function of key press	Yellow	2774	705	0.2541	0.313
	Green	2755	674	0.2446	0.748
	Blue	2761	706	0.2557	0.250
	Red	2742	667	0.2433	0.793
	Pass	1614	375	0.2323	0.953
	All	12646	3127	0.2473	0.763

Randomness in groups of 100 trials:

Chi-sq. = 299.6141 D.F. = 345 Prob. = 0.9628

TABLE 2

FOUR-STATE ELECTRONIC RANDOM NUMBER GENERATOR

SUBJECT	MEAN SCORE/100 TRIALS OVER 2500 TRIALS	BINOMIAL PROBABILITY
S1	25.76	0.22
S2	29.36	3×10^{-7}
S3	24.67 (750 trials)	0.60
S4	25.76	0.22
S5	25.20	0.42
S6	25.40	0.33
S7 (replication)	27.88	4.8×10^{-4}
All trials	26.47 (15750 trials)	1.1×10^{-5}

TABLE 3

MID-EXPERIMENT RANDOMNESS TESTS

	BUTTONS				Number of Trials	Chi-Sq.	Binom. Prob.
	Yellow	Green	Blue	Red			
Initial States	24	29	23	24	100	0.880	>0.80
Transitions Y	204	199	199	216	818	0.944	>0.80
To (From) G	192	223	222	207	844	3.043	>0.30
B	212	207	226	222	867	1.064	>0.70
R	209	207	222	221	859	0.860	>0.80
All States	841	865	892	890	3488	1.988	>0.50
Nondiagonal Transitions	613	613	643	645	2514	1.535	>0.50
Diagonal Transitions	204	223	226	221	874	1.341	>0.70

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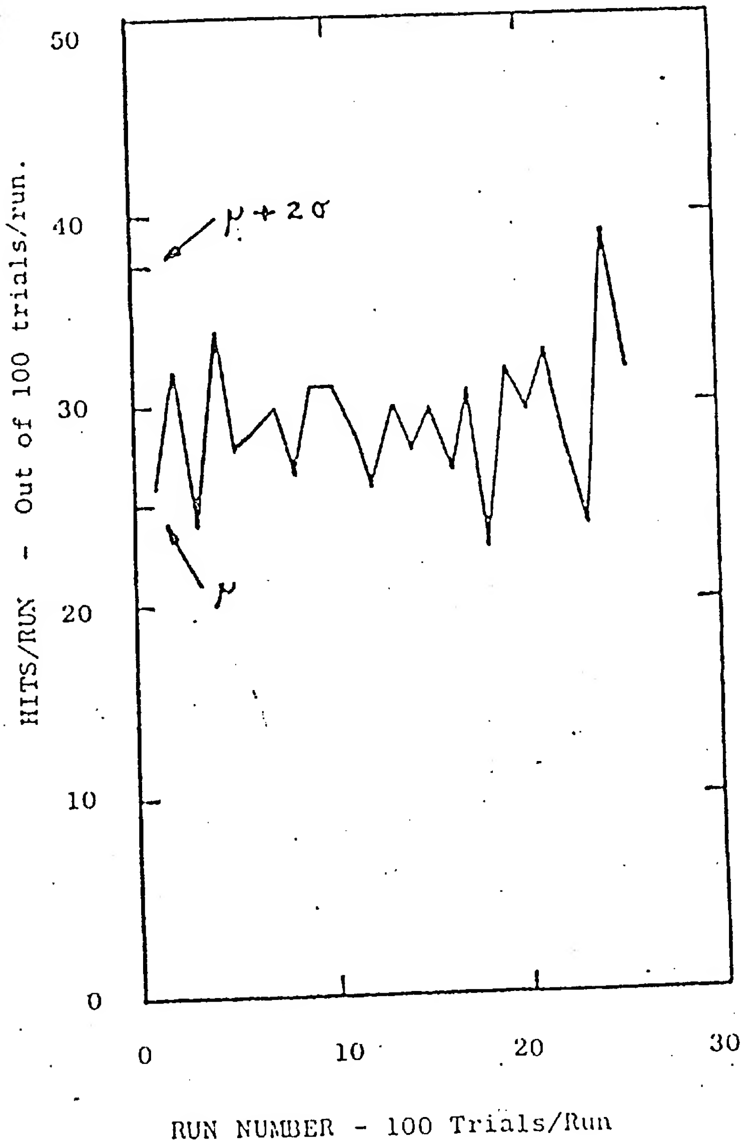

p = ¼ per trial

FIGURE 12 DATA SUMMARY FOR SUBJECT 2

Table

Test	Description	Scoring S1	S2	S3	S4	S5	S6
Halstead Category Test	Nonverbal test requiring abstraction of conceptual relationships. Score: Total errors.	7	14	33	26	6	28
Tactual Performance Test	Requires placement of 10 geometrically shaped blocks in their correct locations on a formboard while blindfolded. Separate RT, LT, and bimanual trials. Score: Total time (min.).	16.4	11.8	7.7	7.7	11.4	6.9
Speech Perception Test	Discrimination of non-word speech sounds. Score: Total errors.	4	2	0	2	5	3
Seashore Rhythm Test	Discrimination of nonverbal rhythms. Score: Number correct.	27	25	28	29	26	29
Finger Tapping Test	Measure of finger oscillation rate for 10-sec. period, both RT and LT hand trials. Score: No. taps/10 sec.	RT/LT 53/50	RT/LT 53/49	RT/LT 48/47	RT/LT 54/53	RT/LT 47/47	RT/LT 48/43
Trail Making Test (Part A)	Requires connecting numbered circles in order from 1 to 25. Paper and pencil task. Score: Total times (sec)	40	16	18	19	30	27
Trail Making Test (Part B)	Requires connecting alphabetic and numbered circles by alternating 1→A→2→B, etc. Score: Total time (sec)	56	50	55	50	54	53
Knox Cube Test	Measure of attention span and immediate visual memory. Score: Number correct.	13	14	13	16	17	17
Raven Progressive Matrices	Nonverbal intelligence test involving spatial matrices. Score: Number correct.	39	53	49	55	60	54
Verbal Concept Attainment Test	Requires abstraction of verbal conceptual relationships. Score: Number correct.	22	24	27	23	21	24
Buschke Memory Test	Requires learning a 20-word list in a maximum of 12 trials with repetition of words omitted after each trial. Score: Max. no. words correctly remembered; List: no. words consistently remembered	Total: 14/20 List: 8/20	17/20 14/20	18/20 11/20	19/20 16/20	20/20 15/20 (8 trials)	20/20 16/20 (7 trials)
Grooved Pegboard Test	Requires insertion of 25 pegs in their holes in a pegboard. Both RT and LT hand trials. Score: Total time (sec).	RT/LT 76/74	RT/LT 69/70	RT/LT 58/67	RT/LT 59/67	RT/LT 72/70	RT/LT 48/50
Spatial Relations Subtest of the PMA	Requires mental rotation and identification of figures rotated in 2 dimensions. Score: no. correct - no. errors.	-	-	-	-	60	52
Gottschaldt Hidden Figures Test	Requires tracing outline of simple figure hidden within lines of more complex figure. Score: Time and no. correct.	Poor	Avg.	-	v.good	outst.	outst.

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